Exercise 4

Solve the differential equation.

$$y'' + y' - 12y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Plug these formulas into the ODE.

$$r^2 e^{rx} + re^{rx} - 12(e^{rx}) = 0$$

Divide both sides by e^{rx}.

$$r^2 + r - 12 = 0$$

Solve for r.

$$(r+4)(r-3) = 0$$

$$r = \{-4, 3\}$$

Two solutions to the ODE are e^{-4x} and e^{3x}. By the principle of superposition, then,

$$y(x) = C_1 e^{-4x} + C_2 e^{3x},$$

where C_1 and C_2 are arbitrary constants.